MATH 590: QUIZ 8 SOLUTIONS

Name:

1. Let $T : V \to V$ be a linear transformation with $\dim(V) = n$. (i) Define what it means for T to be diagonalizable and (ii) State an equivalent condition to the diagonalizability of T. (4 points)

Solution. (i) T is diagonalizable if there exists a basis $\alpha \subseteq V$ such that the matrix of T with respect to α is a diagonal matrix.

(ii) T is diagonalizable if and only if $p_T(x) = (x - \lambda_1)^{e_1} \cdots (x - \lambda_r)^{e_r}$, with distinct $\lambda_i \in F$, and for each λ_i, $\dim(E_{\lambda_i}) = e_i$, i.e., for each eigenvalue λ_i, its geometric multiplicity equals its algebraic multiplicity.

2. Determine whether or not the matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 2 & 1 & 3 \\ 5 & 0 & 1 \end{pmatrix}$ is diagonalizable. You must justify your answer. (6 points).

Solution. $p_A(x) = \begin{vmatrix} x-2 & 0 & 0 \\ -2 & x-1 & -3 \\ -5 & 0 & x-1 \end{vmatrix} = (x-2)(x-1)^2$. Thus, A has eigenvalues 2 and 1, with algebraic multiplicities 1 and 2, respectively.

$E_1 =$ nullspace of $\begin{pmatrix} 2-1 & 0 & 0 \\ 2 & 1-1 & 3 \\ 5 & 0 & 1-1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & 3 \\ 5 & 0 & 0 \end{pmatrix} \xrightarrow{\text{EROs}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}$, which has rank two. Thus, the dimension of the nullspace of this latter matrix is 1. Therefore, $\dim(E_1) = 1 < 2$, showing that A is **not** diagonalizable.